Is consciousness computable?

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Introduction

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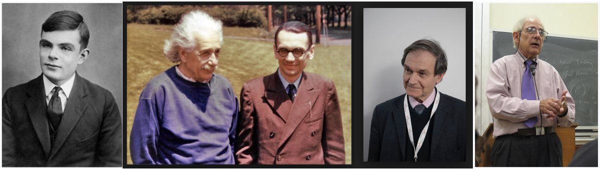
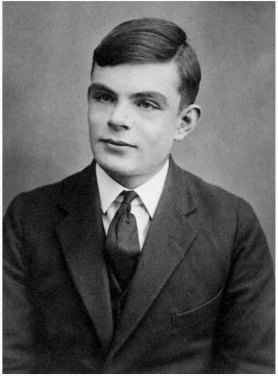


Alan Turing

(1912-1954)

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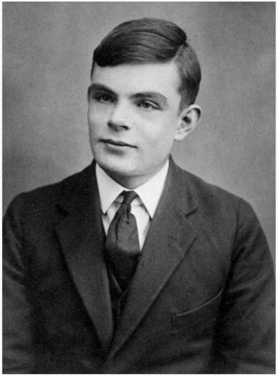
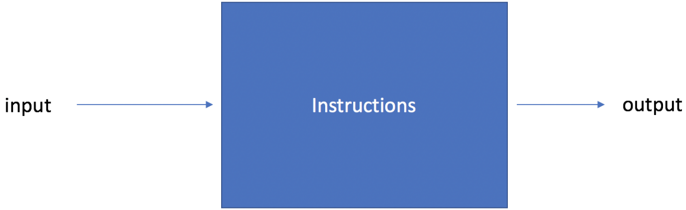
(1912-1954)

Turing is widely considered to be the father of theoretical computer science and artificial intelligence (wikipedia)

Algorithms

A finite set of well defined instructions or rules that given an input it produces an output.



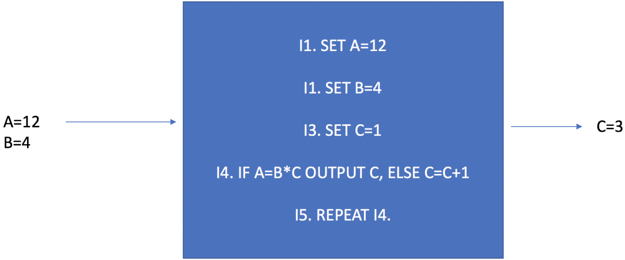
Example of Algorithm
Division Algorithm

Divide 12 by 4

I1. SET A=12
I1. SET B=4
I3. SET C=1
I4. IF A=B*C OUTPUT C, ELSE C=C+1 AND DO I5
I5. REPEAT I4.

Example of Algorithm

Division Algorithm



Decision problem

Is there any algorithm that takes as input a mathematical statement and a list of axioms such that it returns 1 if the statement can be deduced from the axioms and 0 otherwise?

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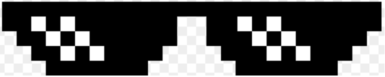

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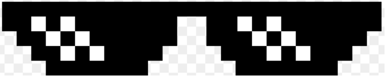

but machines can do great stuff!

What can machines do?

- Beat the chess master Kasparov.
- Beat the Go master Lee Sedol.

What can machines do?

DeepMind

Demis Hassabis

What can machines do?
Chatbots

What can machines do?
Chatbots

- Turing test.
- Loebner Prize.

An argument against
John Searle

A true intelligence must be able to understand (?)

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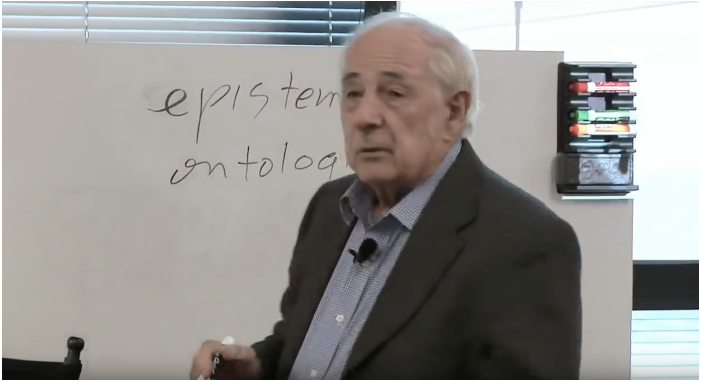

John Searle: "Consciousness in Artificial Intelligence" — Talks at Google (2015)

John Searle

Chinese room

- he proposed a thought experiment: "chinese room".
- program cannot give a computer a "mind", "understanding" or "consciousness".

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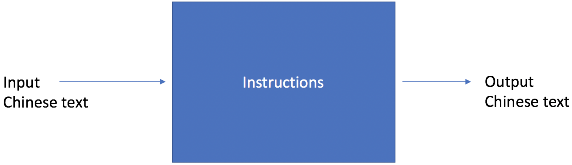


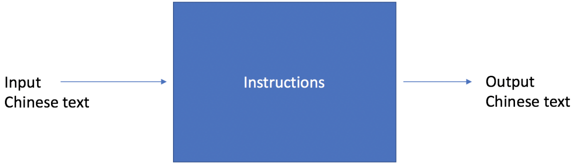
Chinese room

Instructions
(in English)

Input
Chinese text

Output
Chinese text

Observations



- Such algorithm might evoke consciousness.
- The human may eventually learn Chinese.

back to the decision problem

- machines cannot handle the decision problem.
- can we?

An intrinsic property of mathematics
Incompleteness Godel's theorem

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- $0 = \phi$, $1 = \{\phi\}$, $2 = \{\phi, \{\phi\}\}$, $3 = \{\phi, \{\phi, \{\phi\}\}\}$

Zermelo-Fraenkel Axioms

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Ernst Zermelo (1871-1953) gave axioms of set theory, which were improved by Adolf Fraenkel (1891-1965). This system of axioms called ZF or ZFC (if the axiom of choice is included) is the most widely used definition of sets.

(ZF1) Axiom of Extension(ality) Two sets are equal iff they have the same members:

$$\forall a \forall b(\forall x(x \in a \leftrightarrow x \in b) \leftrightarrow a = b)$$

(ZF2) Empty Set Axiom There is a set $\varnothing$ with no elements:

$$\exists a \forall x(\neg(x \in a))$$

(ZF3) Pairing Axiom If a and b are sets, there exists a set, $\{a,b\}$, whose members are exactly a and b:

$$\forall a \forall b \exists c(\forall x(x \in c \leftrightarrow ((x = a) \vee (x = b))))$$

Special case: $\{a,a\}$ denoted $\{a\}$—**singleton set**.

(ZF4) Union Axiom If a is a set there exists a set $\bigcup a$ whose members are the members of members of a:

$$\forall a \exists b \forall x(x \in b \leftrightarrow \exists y((x \in y) \wedge (y \in a)))$$

Notation: $\bigcup\{a,b\}$ is denoted $a \cup b$.

(ZF5) Powerset Axiom If a is a set, there is a set, $\wp(a)$, whose members are the subsets of a:

$$\forall a \exists b \forall x(x \in b \leftrightarrow x \subseteq a)$$

[shorthand: $a \subseteq b$ stands for $\forall x(x \in a \to x \in b)$].

(ZF6) Separation Axiom (Scheme) (*alias* **Subset**, *alias* **Selection**). For any admissible formula $\varphi(x)$ and for any set a there is a set

$$\{x \in a : \varphi(x)\}$$

whose members are those members of a which satisfy $\varphi(x)$.

(ZF7) Axiom of Infinity There exists an inductive set, that is,

$$\exists a(\varnothing \in a \wedge \forall x(x \in a \to x \cup \{x\} \in a))$$

(ZF8) Replacement Axiom (Scheme) 'The range of a partial function whose domain is a set is itself a set'. Let $\varphi(x,y)$ be an admissible formula such that $\forall s \exists t((\varphi(s,t) \wedge \forall u(\varphi(s,u) \to u = t)))$ (that is, φ is a **class function**). Then

$$\forall a \exists b \forall y(y \in b \leftrightarrow \exists x(x \in a \wedge \varphi(x,y)))$$

(that is, when restricted to a set a, the range of φ is a set b).

(ZF9) Axiom of Foundation (*alias* **Regularity**) Every non-empty set is well-founded (that is, contains an element minimal w.r.t. $\in$):

$$\forall a(a \neq \varnothing \to (\exists x(x \in a \wedge x \cap a = \varnothing)).$$

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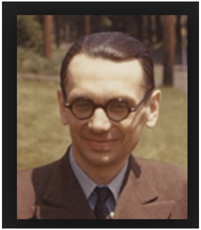
Godel's Theorems



Kurt Godel (1906 – 1978)

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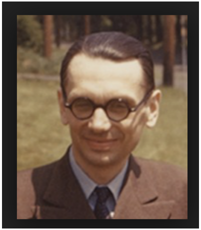

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Q1- Is this set of axioms consistent?

Q2- Are all mathematical statements either probable or dis-probable ?

R1 - (partial answer) If ZFC can prove that ZFC is consistent then it is not consistent. (weird)

R1 - There are theorems that are true that cannot be proven nor disproved.

let's review an example of a similar phenomena!

Example in Set theory

$$\{a,b,c\}$$

$$\left\{ \begin{array}{l} \{\}, \\ \{a\}, \{b\}, \{c\}, \\ \{a,b\}, \{a,c\}, \{b,c\}, \\ \{a,b,c\} \end{array} \right\}$$

Example in Set theory

{a,b,c}

{
{},
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}

Powerset of A , P(A)

A → P(A)

Example in Set theory

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Powerset of A , P(A)

$A \longrightarrow P(A)$

- In general a set with N elements has 2^N subsets.

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Example in Set theory

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$$\mathcal{P}(\mathcal{P}(\mathbb{N}))$$

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CONTINUUM HYPOTHESIS

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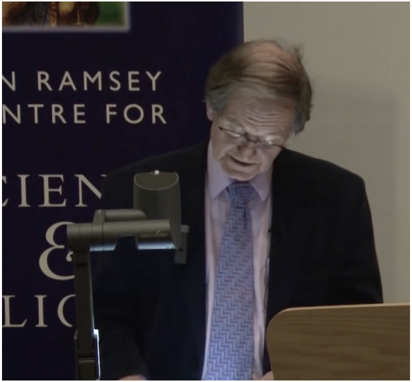
CONTINUUM HYPOTHESIS
(Independent of ZFC)

Penrose's view

...to me this says human understanding in this very limited area (mathematics) [...] does not seem to me something computational because there is no set of rules which can give you what we can achieve with human understanding.



Roger Penrose

Penrose's view

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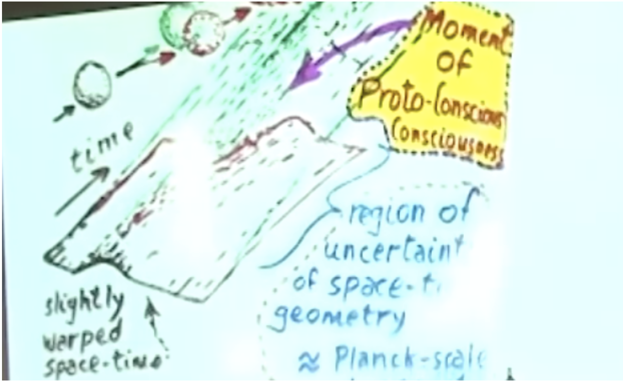

Thank you.